Probabilistic estimation of earthquake growth to a large one in earthquake early warning system: Re-estimation for the Nankai-trough region

T. Iwata[1], M. Imoto[2], S. Horiuchi[2], and Y. Ogata[1]

1: The Institute of Statistical Mathematics, Tokyo
2: National Research Institute for Earth Science and Disaster Prevention

Earthquake Early Warning System (EEWS) in Japan

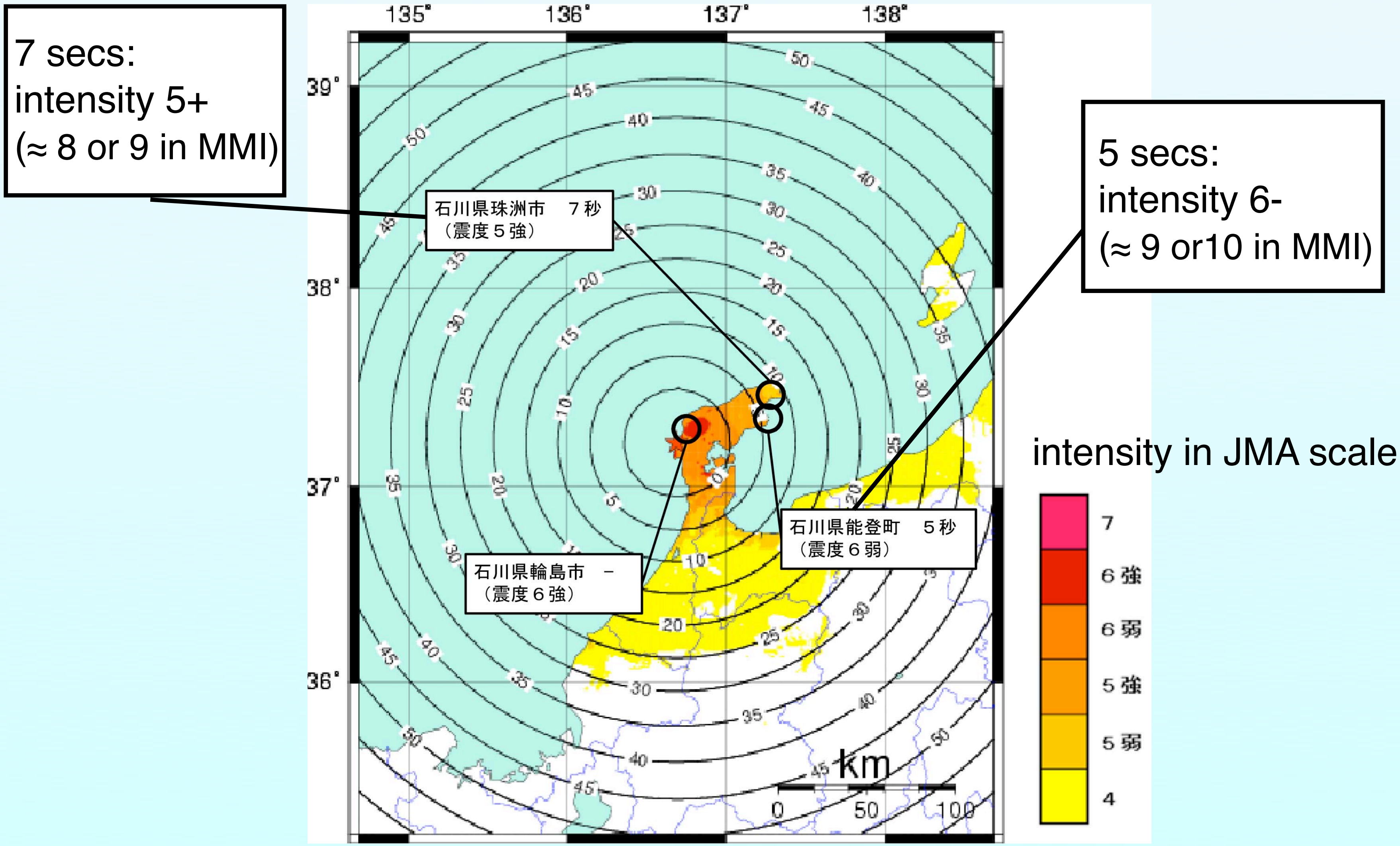
★ Using initial part of *P*-wave, focal parameters (location, magnitude) are determined before arriving of *S*-wave.

★ Japan Meteorological Agency (JMA) started a test run of the EEWS in August 2006.

★ An early warning will be issued broadly in September 2007.

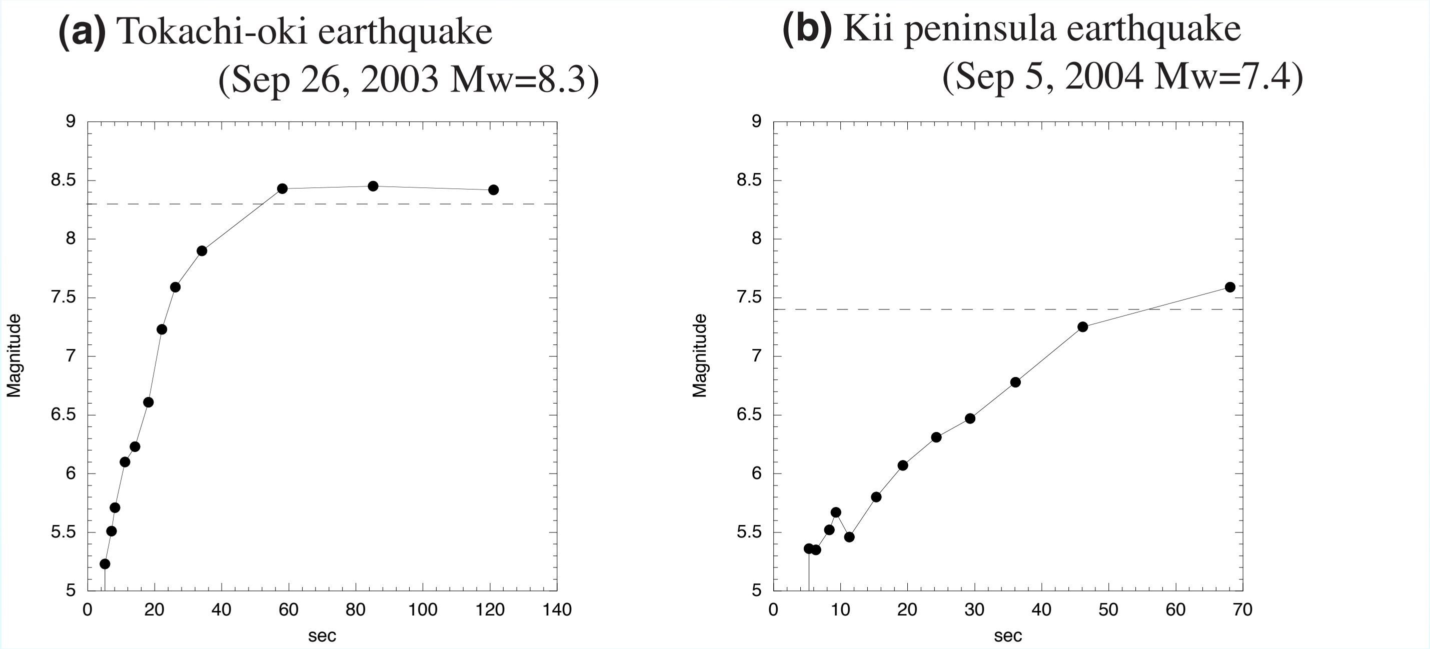
Earthquake Early Warning System (EEWS) in Japan

2007 Noto-Hanto earthquake (March 27, M_{jma}=6.9)



[JMA, 2007]

Estimation of magnitude for large earthquakes



[Negishi, 2004, private communication]

★ The magnitude obtained in **the early stage** of EEWS is **underestimated**.

★ It takes **one minute** to reach “final magnitude”.

★ We **cannot wait** the growing of the magnitude (S-wave must arrive!).

How to distinguish whether or not an earthquake growth to a large one

★ **Predominant period** / **peak amplitude** in the first few seconds of P-wave
[e.g., Olsen & Allen, 2005; Wu & Kanamori, 2005;
Zollo et al., 2006]

↕

argument [e.g., Rydelek & Horiuchi, 2006; Wolfe, 2006]

★ **Magnitude-frequency distribution** (our approach)

→ **probability** of earthquake growth to a large one

Parameters in our approach

M_{fin} : magnitude after rupture completed (final magnitude)
M_{th} : **lower magnitude threshold** of a “large” earthquake
M_{obs} : magnitude **obtained** by EEWS
$p(m)$: probability density function of magnitude
$P(M_{\mathrm{fin}} \geq M_{\mathrm{th}} \mid M_{\mathrm{obs}})$
: conditional probability that M_{fin} is M_{th} or larger
when we obtain M_{obs} (**probability of earthquake growth**)

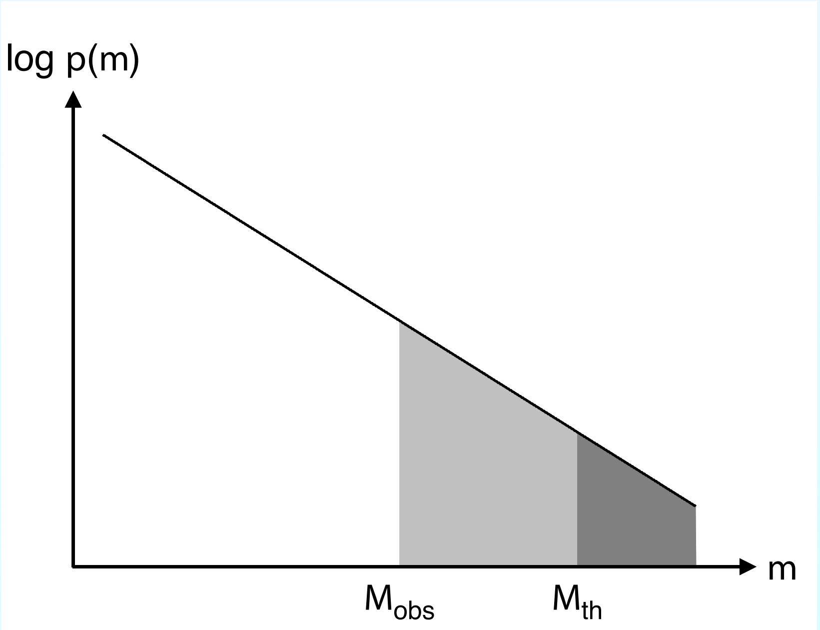
We estimate $P(M_{\mathrm{fin}} \geq M_{\mathrm{th}} \mid M_{\mathrm{obs}})$.

Estimation of $P(M_{\mathrm{fin}} \geq M_{\mathrm{th}} \mid M_{\mathrm{obs}})$

Our assumptions

★ The probability density of M_{fin} follows $p(m)$.

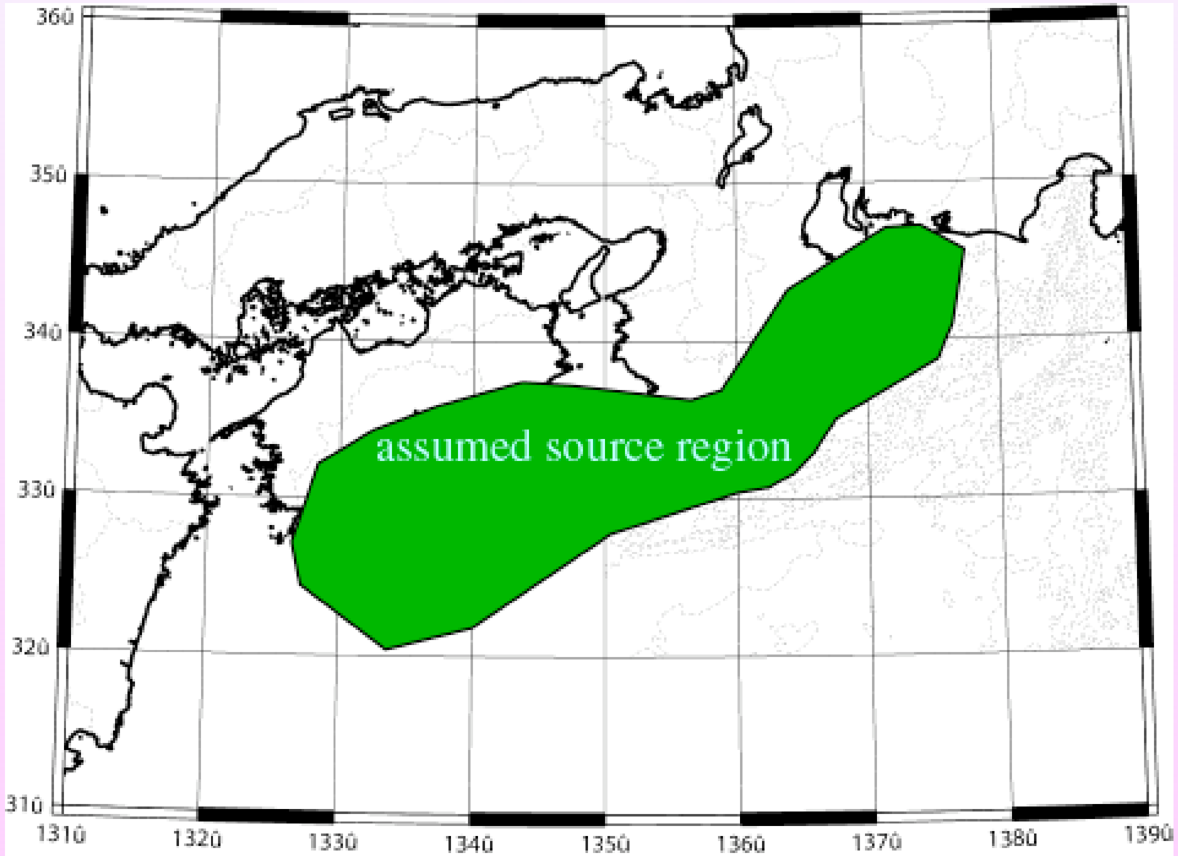
★ M_{fin} is M_{obs} or larger.
(An earthquake only grows up, not grow downward.)



$$P(M_{fin} \geq M_{th} | M_{obs}) = \frac{\int_{M_{th}}^{\infty} p(m)dm}{\int_{M_{obs}}^{\infty} p(m)dm} = \frac{}{ + }$$

If we have $p(m)$, $P(M_{\mathrm{fin}} \geq M_{\mathrm{th}} \mid M_{\mathrm{obs}})$ can be estimated.

Application to the Nankai trough region



[Cabinet office, Government of Japan, 2003]

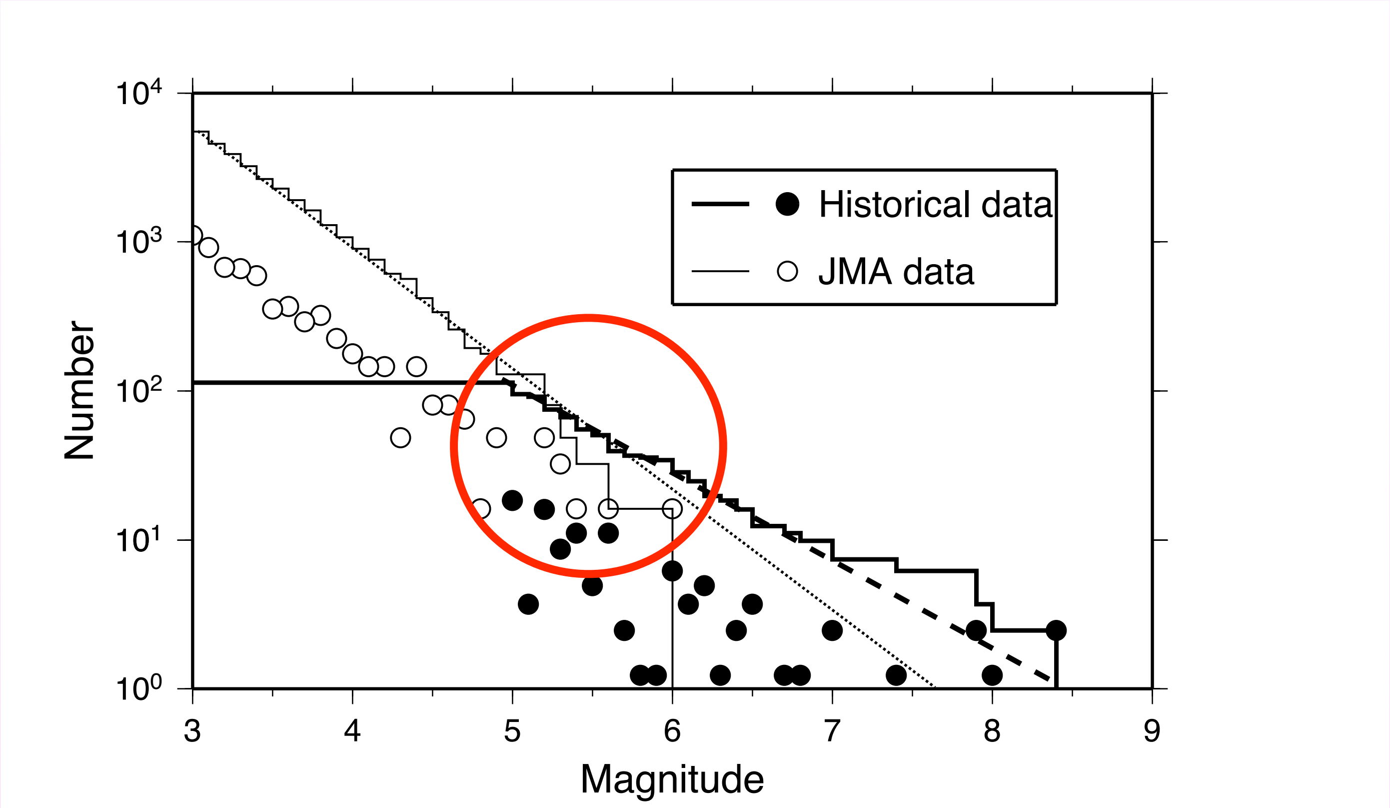
★ Located along the southwestern coast of Japan
★ Plate boundary between the Philippine Sea and Eurasian plates
★ Destructive interplate earthquakes ($M \approx 8$) occurring at intervals of around 100 years

Estimation of $p(m)$, (probability density of magnitude)

★ The estimation is done using the magnitude-frequency of earthquakes occurred in this region previously.

- Historical earthquake data (Usami [1987] & Utsu [1982, 1985])
  + JMA (Japan Meteorological Agency) data from 1600 to 2006
  # of eq : 92

Comparison of the two catalogues

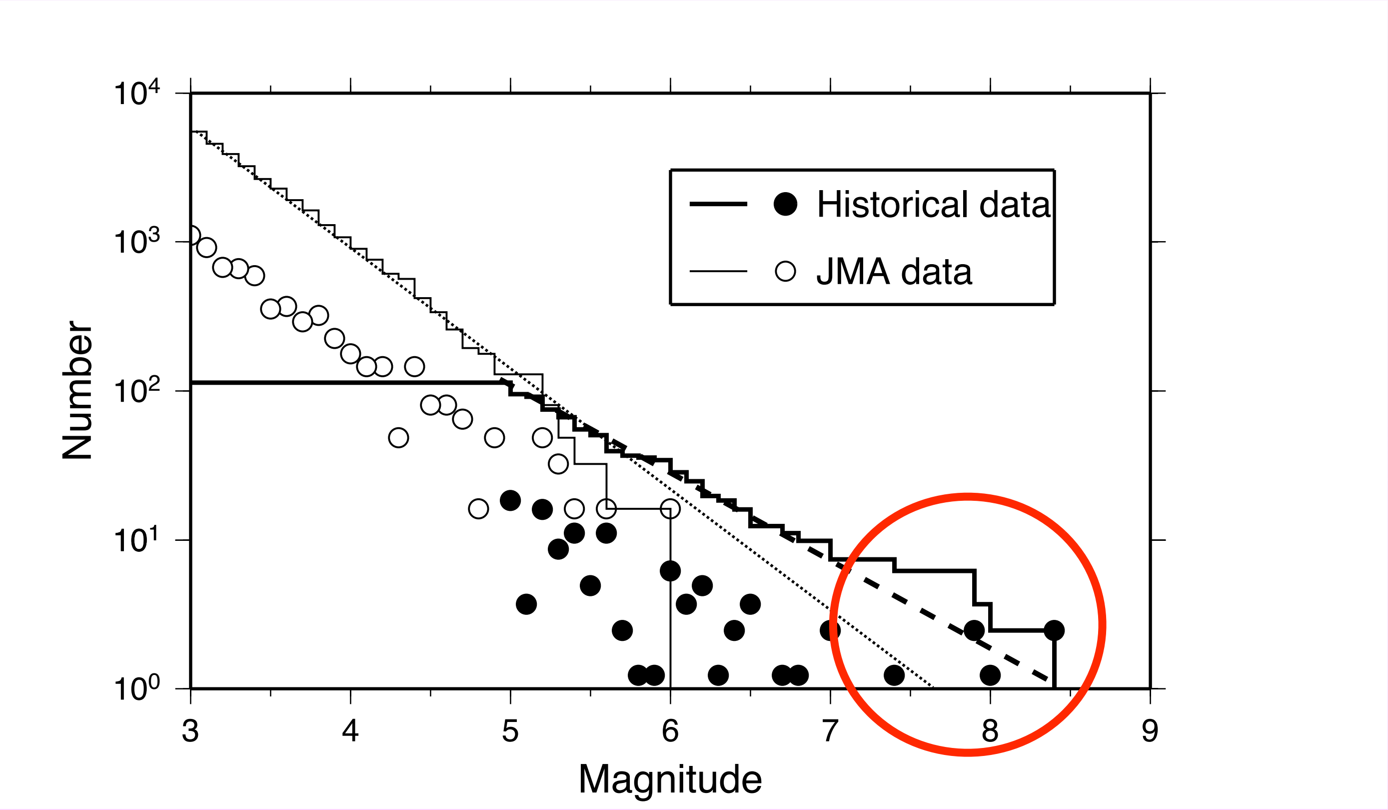


★ The number of earthquakes of magnitude **5.0 or larger** is **almost same** in historical and JMA catalogues

★ A **log-linear** magnitude-frequency relation **holds well** in the historical catalogue

→ The historical catalogue includes a **near-perfect selection** of earthquake of magnitude **5.0 or larger**.

Comparison of the two catalogues



★ The magnitude-frequency of the historical earthquakes deviates from the log-linear relation in the range of large magnitudes (> 7 ?).
→ **characteristic earthquakes**?

Statistical model for the estimation of *p*(*m*)

$$p(m) = (1-\gamma)\cdot \underbrace{\beta \exp(-\beta m)}_{\text{GR}} + \gamma \cdot \underbrace{\frac{1}{\sqrt{2\pi}\sigma} \exp\left(-\frac{(m-\mu)^2}{2\sigma^2}\right)}_{\text{characteristic earthquake}}$$

★ The earthquakes following the **GR law** and **characteristic earthquakes** are **mixed** in the ratio of **1-γ : γ**

★ We consider two models :
model A : $\gamma = 0$ (fixed)   (only **GR**)
model B : $\gamma \neq 0$ (**GR** + **characteristic earthquake**)

Model comparison for small dataset

★ For small dataset, a **Bayesian approach** is used.

★ The marginal likelihood $\mathrm{pr}(\boldsymbol{m}|H_k)$ is estimated instead of the (usual) likelihood.

$$\mathrm{pr}\left(\boldsymbol{m}|H_k\right) = \int_{\Theta_k} L(\theta_k \mid \boldsymbol{m}, H_k)\pi(\theta_k \mid H_k)d\theta_k \quad (k = A, B)$$

$L(\cdot)$: likelihood
θ_k : model parameter
Θ_k : parameter space
H_k : hypotheses for the models
$\pi(\cdot)$: prior distributions

Prior distributions

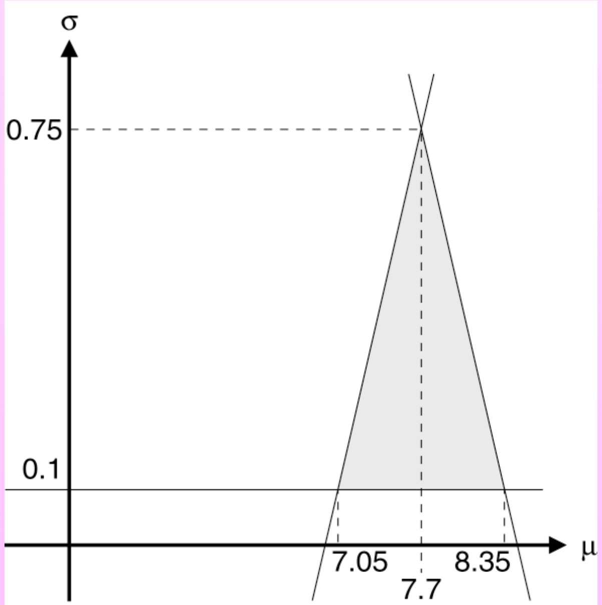
$\pi\ (\beta) = U(0, 5)$ ...... $b < 2$ [e.g., Utsu, 1971]

$\pi\ (\gamma) = U(0, c)$ ...... The ratio of characteristic eq. is less than c.

$\pi\ (\mu) = U(7.05, 8.35)$

$\pi\ (\sigma) = U(0.1, 0.75\text{-}|\mu\text{-}7.7|)$

...... Majority of the magnitude of characteristic earthquakes ($\mu \pm \sigma$) is between 7.0 and 8.4 (the largest magnitude of the dataset), and σ is larger than the interval size (= 0.1).

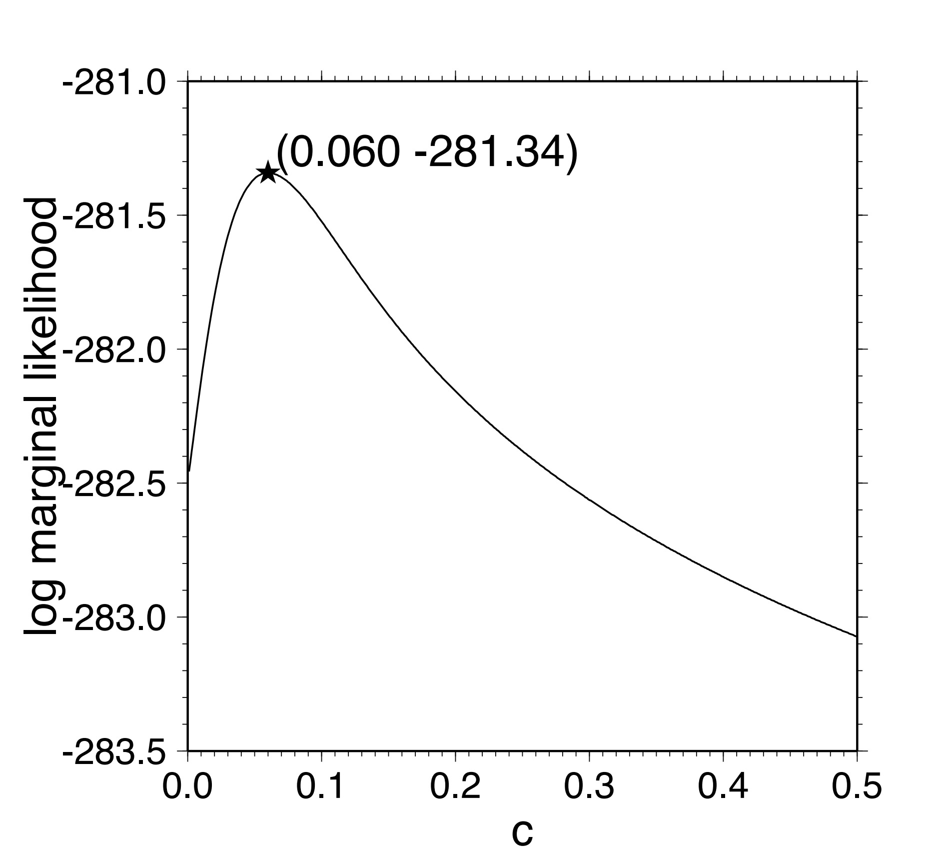


Hyperparameter

c : the parameter included in prior distributions : hyperparameter

★ We choose the value of c which maximizes the marginal likelihood $\mathrm{pr}(\boldsymbol{m}|H_\mathrm{k})$.

→ $c = 0.060$



Model selection / Parameter estimation

★ABIC (Akaike Bayesian Information Criterion)

ABIC= -2 x (maximum ln pr($\boldsymbol{m}|H_k$)) + 2 x (# of hypermaremeter)
(**smaller** ABIC → **better** model)

★ $Q_k(\theta_k\,|\,\boldsymbol{m})$: probability density of the parameters
(posterior distribution)

$$Q(\theta_k\,|\,\boldsymbol{m}) = \frac{L(\theta_k\ \boldsymbol{m}, H_k)\pi(\theta_k\,|\,H_k)}{\int_{\Theta_k} L(\theta_k\,|\,\boldsymbol{m}, H_k)\pi(\theta_k\,|\,H_k)d\theta_k}$$

★ $\hat{\theta}_k$: Bayes estimate of the model parameters (posterior mean)

$$\hat{\theta}_k = \int_{\Theta_k} \theta_k Q(\theta_k\,|\,\boldsymbol{m})d\theta_k$$

Results of the estimation

model	$\hat{\beta}$	$\hat{\gamma}$	$\hat{\sigma}$	$\hat{\mu}$	ABIC
A	1.31	0.0(restricted)	—	—	$\boxed{564.98}$
B	1.47	0.0346	0.281	7.99	$\boxed{564.68}$

★ Model B is better than model A.

★ However, the difference of ABIC is **only 0.3**.

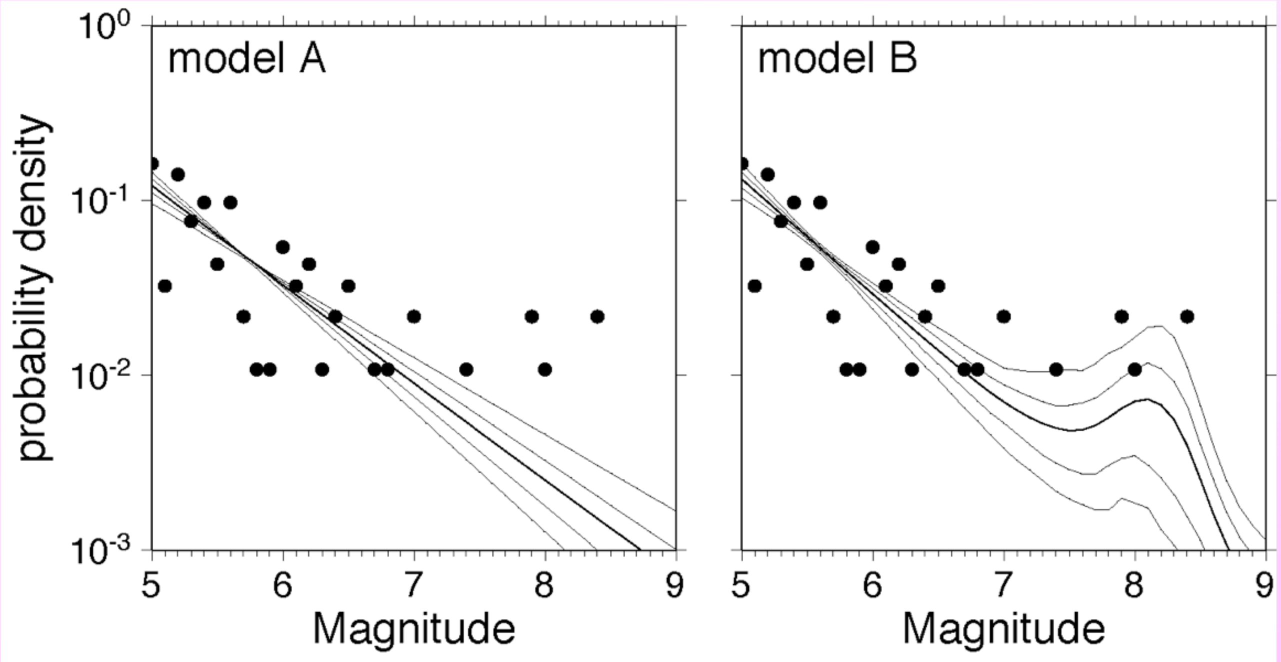
★ Is model B is **significantly better** ?

Estimation of probability density of *m*

★ $\hat{p}(m)$: Bayes estimate of the probability density of m (posterior mean)

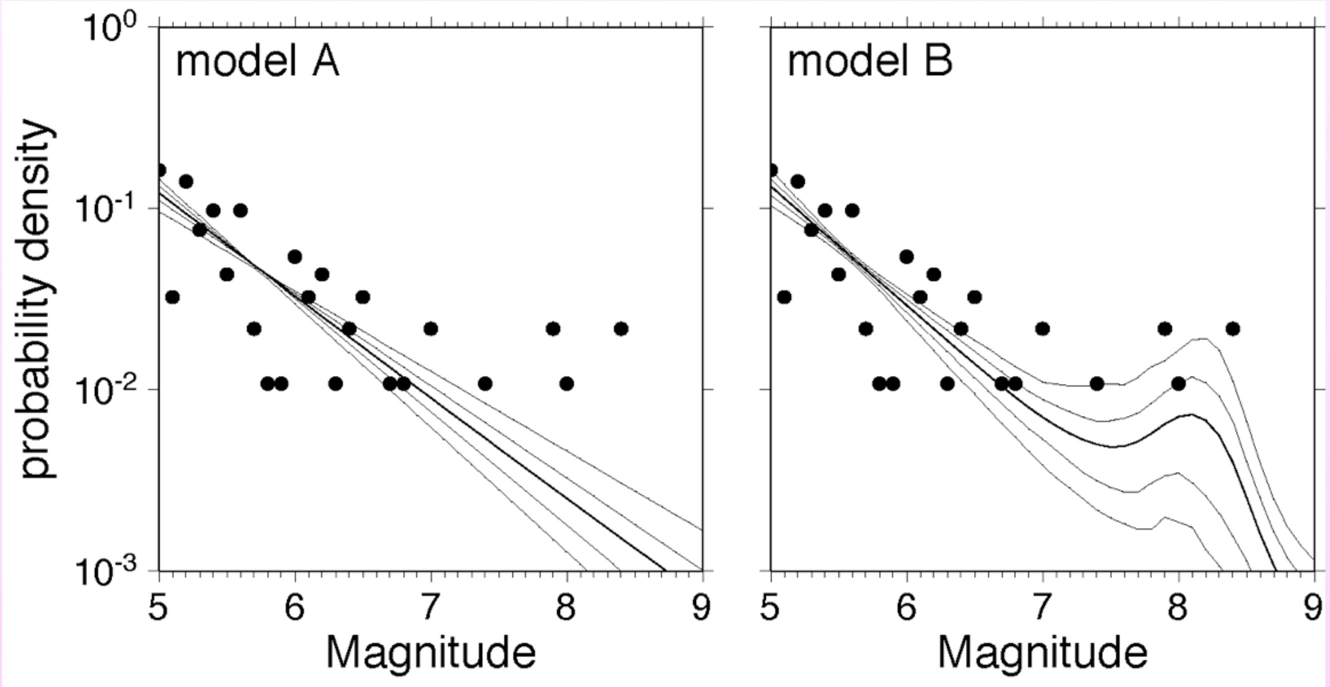
$$\boxed{\hat{p}(m) = \int_{\Theta_k} p(m \mid \theta_k) Q(\theta_k \mid \boldsymbol{m}) d\theta_k}$$

\# 500 samples of parameters from $Q(\theta_k \mid \boldsymbol{m})$ are generated using the rejection method [e.g., Press et al., 1992]



The estimated $p(m)$ is **much different** in models A and B.

Estimation of probability density of *m*



The estimated $p(m)$ is **much different** in models A and B.

$P(M_{fin} \geq M_{th} \mid M_{obs})$ must be **much different**.

For practical use of EEWS, we should determine the unique $P(M_{fin} \geq M_{th} \mid M_{obs})$, but ......

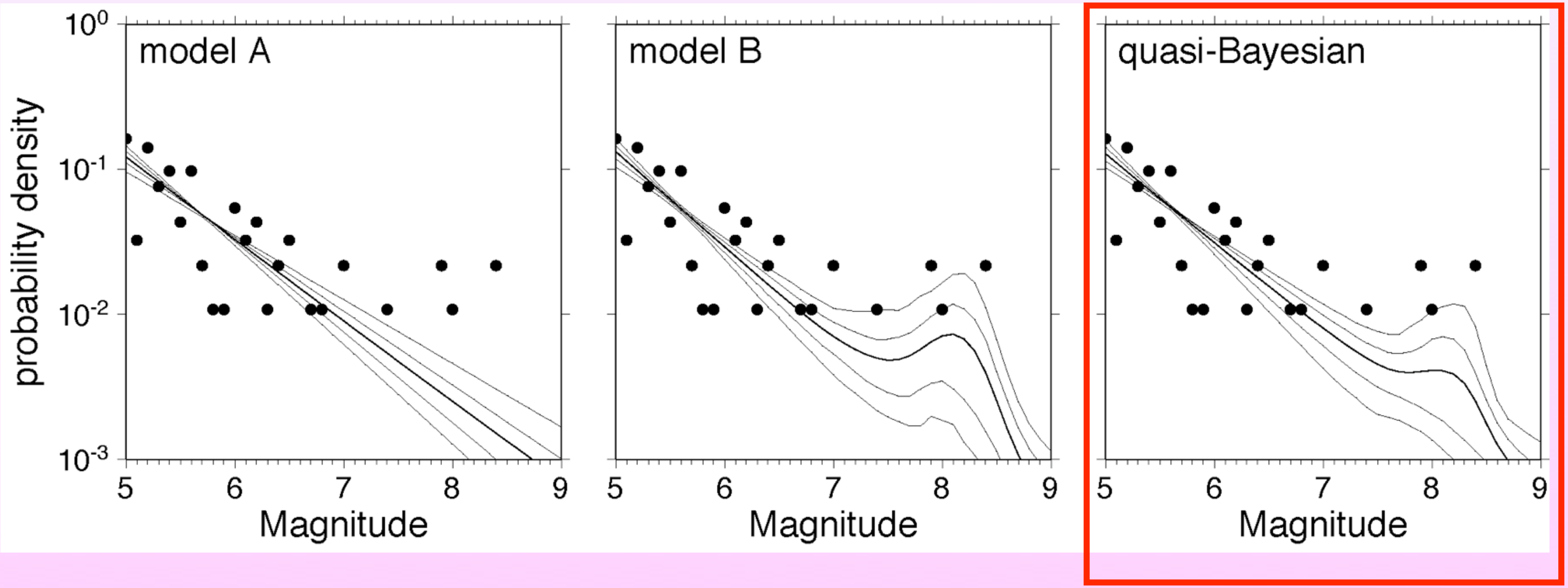
quasi-Bayesian method

★ suggested by Akaike [1979, 1980]
★ **mixture** of the various **competitive** models
★ the ratio (probability) of mixture **based on AIC / ABIC**
★ the probability of selecting the *k*-th model :

$$q_k = \frac{\exp(-\mathrm{ABIC}_k / 2)}{\sum \exp(-\mathrm{ABIC}_k / 2)}$$

★ **do not need to choose** the specified model

Results using quasi-Bayesian method



We adopt this.

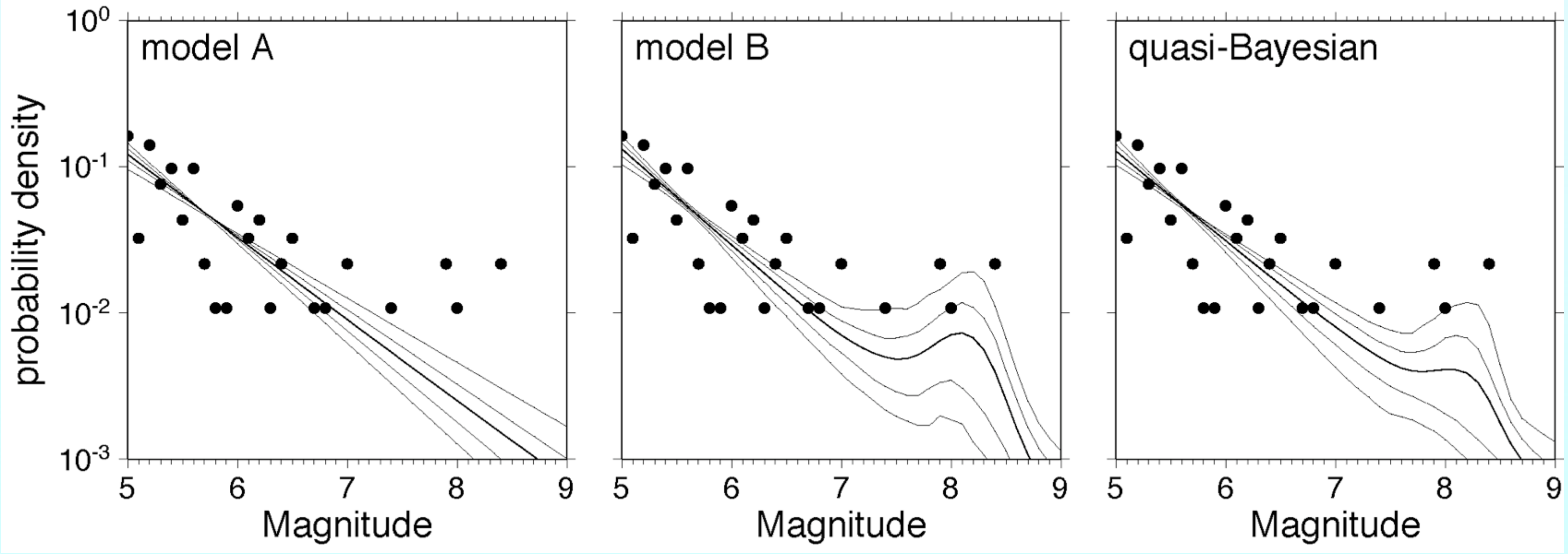
Estimation of $P(M_{\mathrm{fin}} \geq M_{\mathrm{th}} \mid M_{\mathrm{obs}})$

★ $M_{\mathrm{th}} = 7.5$ The destructive eq. in the Nankai trough region is approximately magnitude 8.

★ $\hat{P}(M_{\mathrm{fin}} > 7.5 \mid M_{\mathrm{obs}})$:

Bayes estimate of the probability $P(M_{\mathrm{fin}} > 7.5 \mid M_{\mathrm{obs}})$ (posterior mean)

$$\hat{P}(M_{\mathrm{fin}} > 7.5 \mid M_{\mathrm{obs}}) = \int_{\Theta_k} P(M_{\mathrm{fin}} > 7.5 \mid M_{\mathrm{obs}}) Q(\theta_k \mid \boldsymbol{m}) d\theta_k$$



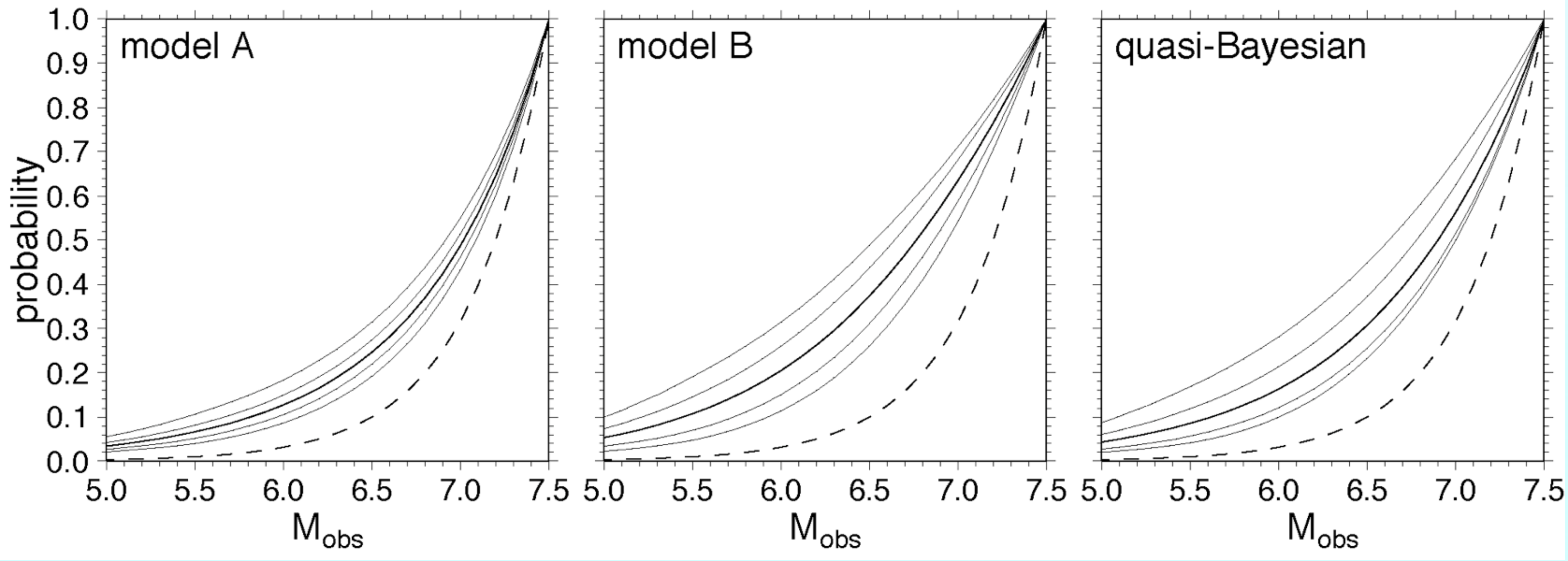
Estimation of $P(M_{\mathrm{fin}} \geq M_{\mathrm{th}} \mid M_{\mathrm{obs}})$

★ $M_{\mathrm{th}} = 7.5$ The destructive eq. in the Nankai trough region is approximately magnitude 8.

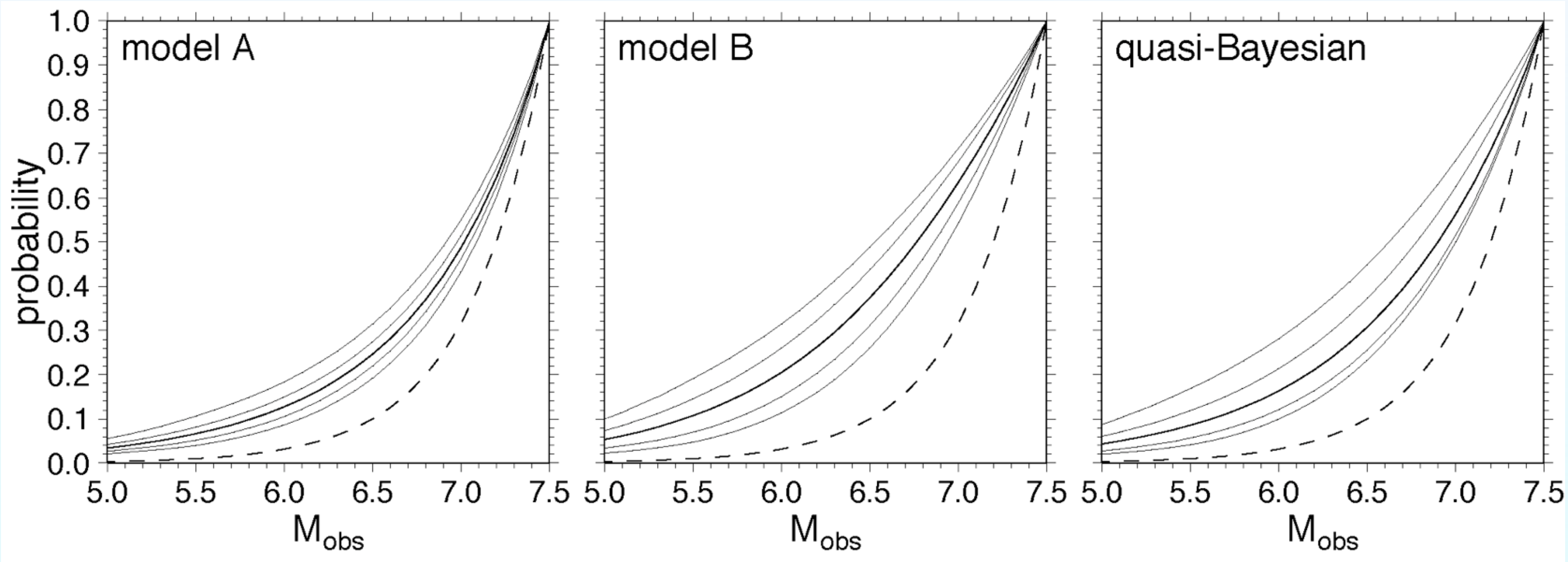
★ $\hat{P}(M_{\mathrm{fin}} > 7.5 \mid M_{\mathrm{obs}})$:

Bayes estimate of the probability $P(M_{\mathrm{fin}} > 7.5 \mid M_{\mathrm{obs}})$ (posterior mean)

$$\hat{P}(M_{\mathrm{fin}} > 7.5 \mid M_{\mathrm{obs}}) = \int_{\Theta_k} P(M_{\mathrm{fin}} > 7.5 \mid M_{\mathrm{obs}}) Q(\theta_k \mid \boldsymbol{m}) d\theta_k$$



Estimation of $P(M_{fin} \geq M_{th} \mid M_{obs})$



★ For example, if $M_{obs} = 6.5$

$$\hat{P}(M_{fin} > 7.5 \mid 6.5) = 0.25 \text{ (model A)}$$

$$0.37 \text{ (model B)}$$

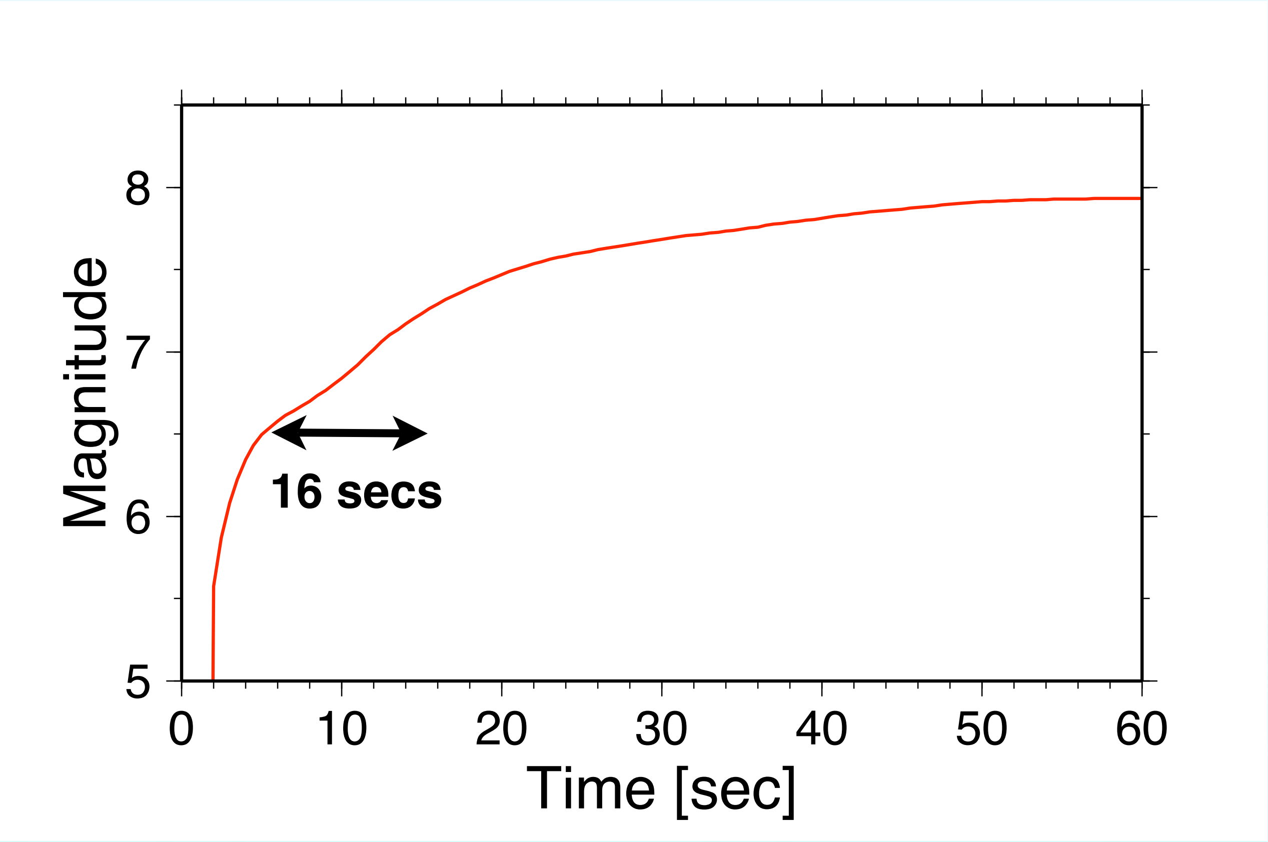
$$\boxed{0.30 \text{ (quasi-Bayesian)}}$$

Estimation of $P(M_{fin} \geq M_{th} \mid M_{obs})$

M 6.5 → 7.5

★ 16 secs [Kikuchi et al., 2003] (1944 Tonankai eq.)

★ 10 -- 15 secs (M8 earthquakes in EEWS)



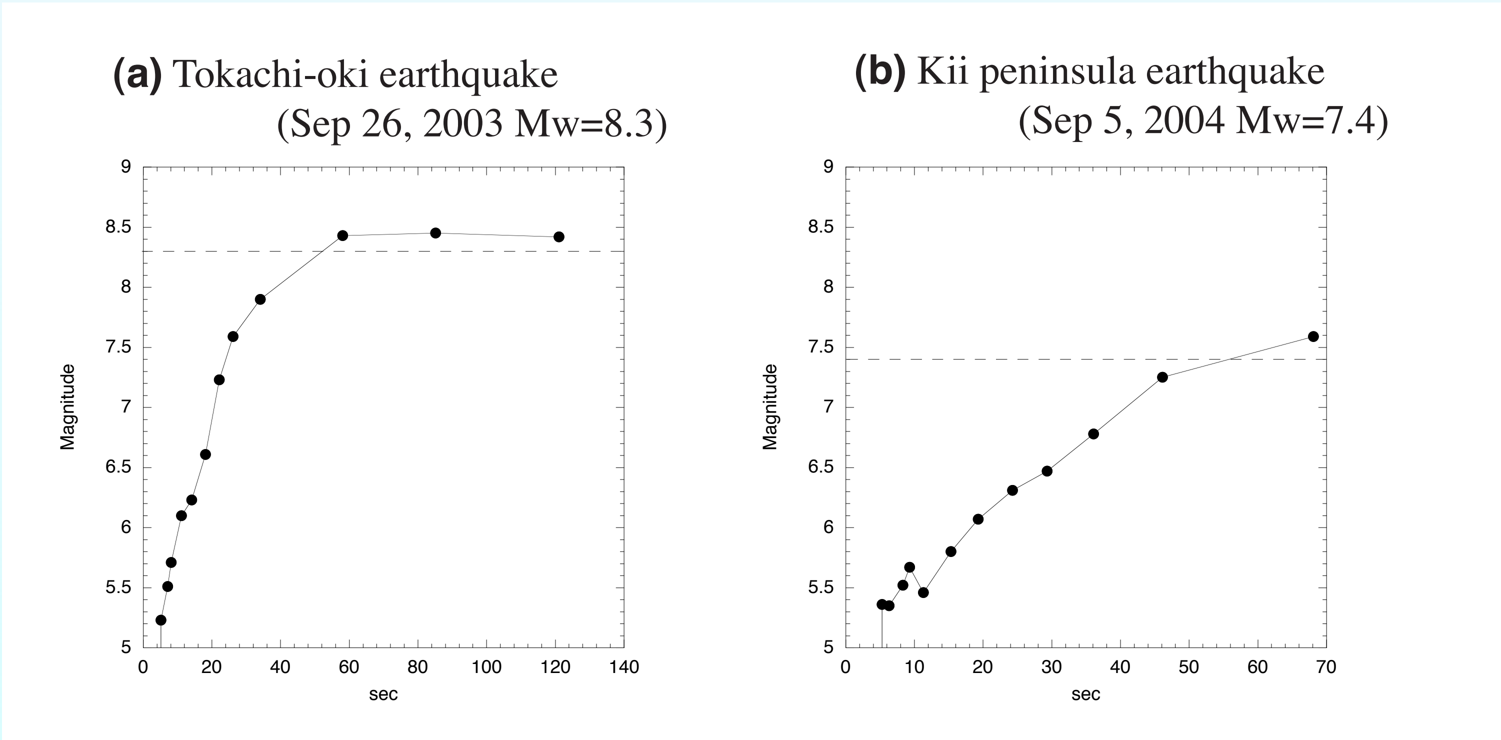
[derived from Kikuchi et al., 2003]

Estimation of $P(M_{fin} \geq M_{th} \mid M_{obs})$

M 6.5 → 7.5

★ 16 secs [Kikuchi et al., 2003] (1944 Tonankai eq.)

★ 10 secs (other M8 earthquakes)



We could issue a **probabilistic** magnitude alarm (10-15 secs) **earlier** than a current deterministic magnitude alarm.

Summary of this talk

★ We suggest a method to estimate the probability of earthquake growth to a large one.

★ The probability is estimated based on the magnitude-frequency distribution.

★ We apply this method to the Nankai trough region.

★ In the estimation of magnitude-frequency distribution, we consider two models.

★ Since the goodness-of-fit of the two models is competitive, quasi-Bayesian method is introduced to determine the probability.

-- For detail:

Iwata, T., Imoto, M., and Horiuchi, S., Probabilistic estimation of earthquake growth to a catastrophic one, Geophys. Res. Lett., 32, L19307, doi:10.1029/2005GL023928.